9.4 Special Right Triangles

What you should learn

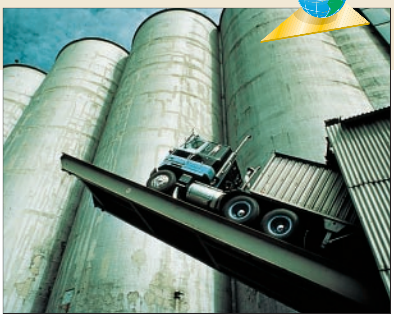
GOAL 1 Find the side lengths of special right triangles.

GOAL 2 Use special right triangles to solve **real-life** problems, such as finding the side lengths of the triangles in the spiral quilt design in **Exs. 31–34**.

Why you should learn it

▼ To use special right triangles to solve **real-life** problems, such as finding the height of a tipping platform in **Example 4**.

REAL LIFE



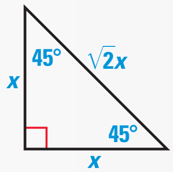
GOAL 1 SIDE LENGTHS OF SPECIAL RIGHT TRIANGLES

Right triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called **special right triangles**. In the Activity on page 550, you may have noticed certain relationships among the side lengths of each of these special right triangles. The theorems below describe these relationships. Exercises 35 and 36 ask you to prove the theorems.

THEOREMS ABOUT SPECIAL RIGHT TRIANGLES

THEOREM 9.8 ***45°-45°-90° Triangle Theorem***

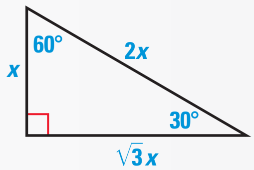
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



Hypotenuse $= \sqrt{2} \cdot$ leg

THEOREM 9.9 ***30°-60°-90° Triangle Theorem***

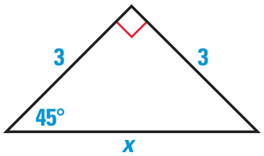
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Hypotenuse $= 2 \cdot$ shorter leg
Longer leg $= \sqrt{3} \cdot$ shorter leg

EXAMPLE 1 *Finding the Hypotenuse in a 45°-45°-90° Triangle*

Find the value of x.



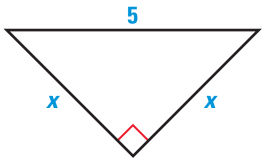
SOLUTION

By the Triangle Sum Theorem, the measure of the third angle is 45°. The triangle is a 45°-45°-90° right triangle, so the length x of the hypotenuse is $\sqrt{2}$ times the length of a leg.

Hypotenuse $= \sqrt{2} \cdot$ leg	**45°-45°-90° Triangle Theorem**
$x = \sqrt{2} \cdot 3$	**Substitute.**
$x = 3\sqrt{2}$	**Simplify.**

EXAMPLE 2 *Finding a Leg in a 45°-45°-90° Triangle*

Find the value of x.



SOLUTION

Because the triangle is an isosceles right triangle, its base angles are congruent. The triangle is a 45°-45°-90° right triangle, so the length of the hypotenuse is $\sqrt{2}$ times the length x of a leg.

$$\text{Hypotenuse} = \sqrt{2} \cdot \text{leg} \qquad \textbf{45°-45°-90° Triangle Theorem}$$

$$5 = \sqrt{2} \cdot x \qquad \textbf{Substitute.}$$

$$\frac{5}{\sqrt{2}} = \frac{\sqrt{2}x}{\sqrt{2}} \qquad \textbf{Divide each side by } \sqrt{2}.$$

$$\frac{5}{\sqrt{2}} = x \qquad \textbf{Simplify.}$$

$$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = x \qquad \textbf{Multiply numerator and denominator by } \sqrt{2}.$$

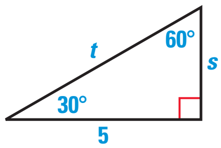
$$\frac{5\sqrt{2}}{2} = x \qquad \textbf{Simplify.}$$

EXAMPLE 3 *Side Lengths in a 30°-60°-90° Triangle*

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

Find the values of s and t.



SOLUTION

Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length s of the shorter leg.

$$\text{Longer leg} = \sqrt{3} \cdot \text{shorter leg} \qquad \textbf{30°-60°-90° Triangle Theorem}$$

$$5 = \sqrt{3} \cdot s \qquad \textbf{Substitute.}$$

$$\frac{5}{\sqrt{3}} = \frac{\sqrt{3} \cdot s}{\sqrt{3}} \qquad \textbf{Divide each side by } \sqrt{3}.$$

$$\frac{5}{\sqrt{3}} = s \qquad \textbf{Simplify.}$$

$$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{5}{\sqrt{3}} = s \qquad \textbf{Multiply numerator and denominator by } \sqrt{3}.$$

$$\frac{5\sqrt{3}}{3} = s \qquad \textbf{Simplify.}$$

The length t of the hypotenuse is twice the length s of the shorter leg.

$$\text{Hypotenuse} = 2 \cdot \text{shorter leg} \qquad \textbf{30°-60°-90° Triangle Theorem}$$

$$t = 2 \cdot \frac{5\sqrt{3}}{3} \qquad \textbf{Substitute.}$$

$$t = \frac{10\sqrt{3}}{3} \qquad \textbf{Simplify.}$$

GOAL 2 USING SPECIAL RIGHT TRIANGLES IN REAL LIFE

EXAMPLE 4 *Finding the Height of a Ramp*

TIPPING PLATFORM A tipping platform is a ramp used to unload trucks, as shown on page 551. How high is the end of an 80 foot ramp when it is tipped by a 30° angle? by a 45° angle?

SOLUTION

When the angle of elevation is 30°, the height h of the ramp is the length of the shorter leg of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ — 30°-60°-90° Triangle Theorem

$40 = h$ — Divide each side by 2.

When the angle of elevation is 45°, the height of the ramp is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = \sqrt{2} \cdot h$ — 45°-45°-90° Triangle Theorem

$\frac{80}{\sqrt{2}} = h$ — Divide each side by $\sqrt{2}$.

$56.6 \approx h$ — Use a calculator to approximate.

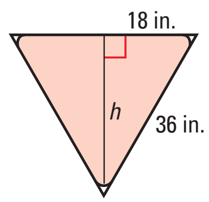
▶ When the angle of elevation is 30°, the ramp height is 40 feet. When the angle of elevation is 45°, the ramp height is about 56 feet 7 inches.

EXAMPLE 5 *Finding the Area of a Sign*

ROAD SIGN The road sign is shaped like an equilateral triangle. Estimate the area of the sign by finding the area of the equilateral triangle.

SOLUTION

First find the height h of the triangle by dividing it into two 30°-60°-90° triangles. The length of the longer leg of one of these triangles is h. The length of the shorter leg is 18 inches.



$h = \sqrt{3} \cdot 18 = 18\sqrt{3}$ — 30°-60°-90° Triangle Theorem

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)\left(18\sqrt{3}\right) \approx 561.18$$

▶ The area of the sign is about 561 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is meant by the term *special right triangles*?

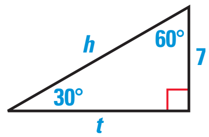
Concept Check ✓

2. **CRITICAL THINKING** Explain why any two 30°-60°-90° triangles are similar.

Use the diagram to tell whether the equation is *true* or *false*.

3. $t = 7\sqrt{3}$ **4.** $t = \sqrt{3h}$ **5.** $h = 2t$

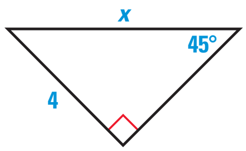
6. $h = 14$ **7.** $7 = \frac{h}{2}$ **8.** $7 = \frac{t}{\sqrt{3}}$



Skill Check ✓

Find the value of each variable. Write answers in simplest radical form.

9.



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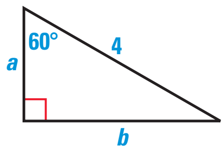

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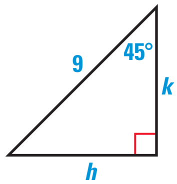


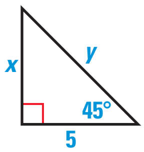
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on p. 820.

USING ALGEBRA **Find the value of each variable. Write answers in simplest radical form.**

12.



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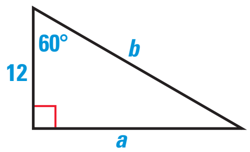

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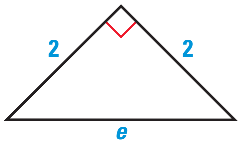

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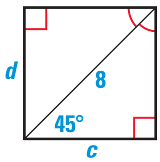

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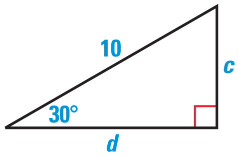

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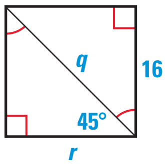

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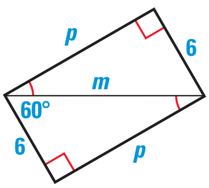

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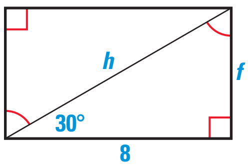

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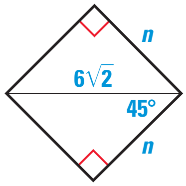


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 12–23
Example 2: Exs. 12–23
Example 3: Exs. 12–23
Example 4: Exs. 28–29, 34
Example 5: Exs. 24–27

FINDING LENGTHS **Sketch the figure that is described. Find the requested length. Round decimals to the nearest tenth.**

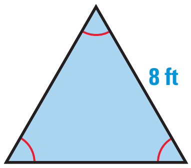
21. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude of the triangle.

22. The perimeter of a square is 36 inches. Find the length of a diagonal.

23. The diagonal of a square is 26 inches. Find the length of a side.

FINDING AREA **Find the area of the figure. Round decimal answers to the nearest tenth.**

24.



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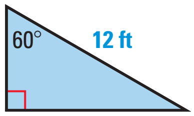

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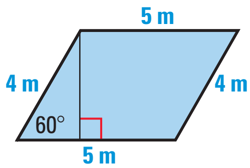
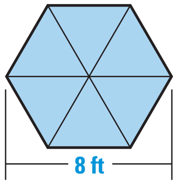


27. **AREA OF A WINDOW** A hexagonal window consists of six congruent panes of glass. Each pane is an equilateral triangle. Find the area of the entire window.

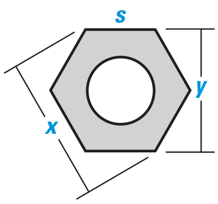


JEWELRY **Estimate the length *x* of each earring.**

28.

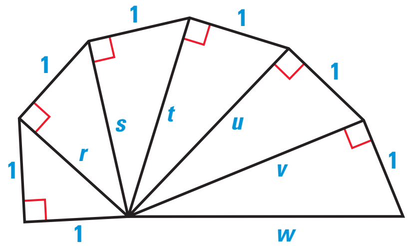
29.

30. **TOOLS** Find the values of x and y for the hexagonal nut shown at the right when $s = 2$ centimeters. (*Hint:* In Exercise 27 above, you saw that a regular hexagon can be divided into six equilateral triangles.)



LOGICAL REASONING **The quilt design in the photo is based on the pattern in the diagram below. Use the diagram in Exercises 31–34.**

Wheel of Theodorus



31. Find the values of r, s, t, u, v, and w. Explain the procedure you used to find the values.

32. Which of the triangles, if any, is a 45°-45°-90° triangle?

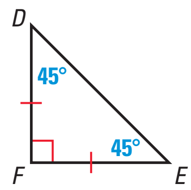
33. Which of the triangles, if any, is a 30°-60°-90° triangle?

34. **USING ALGEBRA** Suppose there are n triangles in the spiral. Write an expression for the hypotenuse of the nth triangle.

35. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 9.8 on page 551.

GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

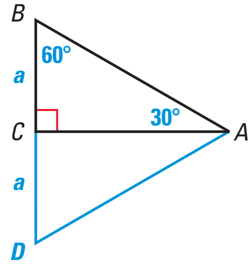


36. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 9.9 on page 551.

GIVEN ▶ $\triangle ABC$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Plan for Proof Construct $\triangle ADC$ congruent to $\triangle ABC$. Then prove that $\triangle ABD$ is equilateral. Express the lengths AB and AC in terms of a.



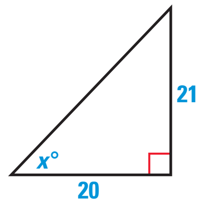
Test Preparation

37. **MULTIPLE CHOICE** Which of the statements below is true about the diagram at the right?

(A) $x < 45$ (B) $x = 45$

(C) $x > 45$ (D) $x \leq 45$

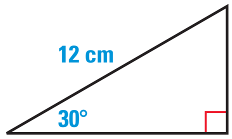
(E) Not enough information is given to determine the value of x.



38. **MULTIPLE CHOICE** Find the perimeter of the triangle shown at the right to the nearest tenth of a centimeter.

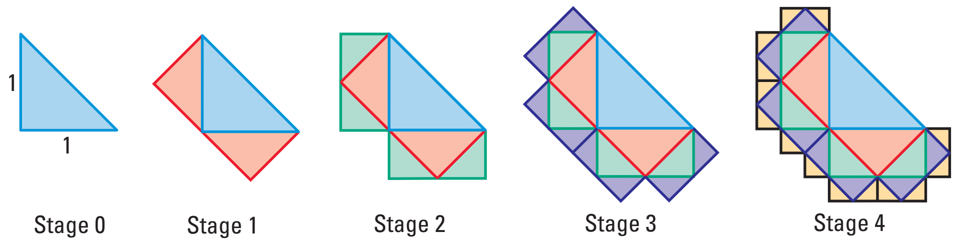
(A) 28.4 cm (B) 30 cm

(C) 31.2 cm (D) 41.6 cm



★ Challenge

VISUAL THINKING In Exercises 39–41, use the diagram below. Each triangle in the diagram is a 45°-45°-90° triangle. At Stage 0, the legs of the triangle are each 1 unit long.



39. Find the exact lengths of the legs of the triangles that are added at each stage. Leave radicals in the denominators of fractions.

40. Describe the pattern of the lengths in Exercise 39.

41. Find the length of a leg of a triangle added in Stage 8. Explain how you found your answer.

EXTRA CHALLENGE
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MIXED REVIEW

42. FINDING A SIDE LENGTH A triangle has one side of 9 inches and another of 14 inches. Describe the possible lengths of the third side. (Review 5.5)

FINDING REFLECTIONS Find the coordinates of the reflection without using a coordinate plane. (Review 7.2)

43. $Q(-1, -2)$ reflected in the x-axis

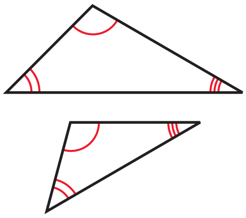
44. $P(8, 3)$ reflected in the y-axis

45. $A(4, -5)$ reflected in the y-axis

46. $B(0, 10)$ reflected in the x-axis

DEVELOPING PROOF Name a postulate or theorem that can be used to prove that the two triangles are similar. (Review 8.5 for 9.5)

47.



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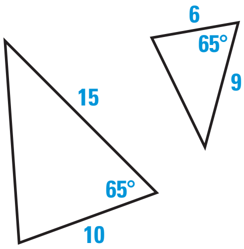

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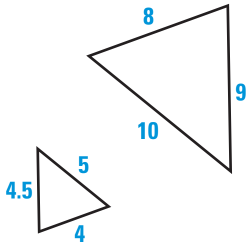


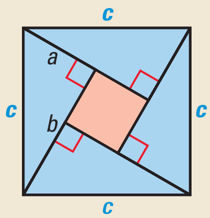
MATH & History — Pythagorean Theorem Proofs

APPLICATION LINK www.mcdougallittell.com

THEN **AROUND THE SIXTH CENTURY B.C.,** the Greek mathematician Pythagoras founded a school for the study of philosophy, mathematics, and science. Many people believe that an early proof of the Pythagorean Theorem came from this school.

NOW **TODAY,** the Pythagorean theorem is one of the most famous theorems in geometry. More than 100 different proofs now exist.

The diagram is based on one drawn by the Hindu mathematician Bhāskara (1114–1185). The four blue right triangles are congruent.



1. Write an expression in terms of a and b for the combined areas of the blue triangles. Then write an expression in terms of a and b for the area of the small red square.
2. Use the diagram to show that $a^2 + b^2 = c^2$. (*Hint:* This proof of the Pythagorean Theorem is similar to the one in Exercise 37 on page 540.)

c. 529 B.C.

School of Pythagoras is founded.

Chinese manuscript includes a diagram that can be used to prove the theorem.

c. A.D. 275

1876

Future U.S. President Garfield discovers a proof of the theorem.

Nicaraguan stamp commemorates the Pythagorean Theorem.

1971